Lower Sixth AL Maths Homework 3 **Name:** Answers

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx} = 12x^3 - 24x^2$

(ii) $\dfrac{d^2y}{dx^2} = 36x^2 - 48x$ (3)

(b) Verify that C has a stationary point when $x = 2$ (2)

Stationary points $\Rightarrow \dfrac{dy}{dx} = 0$

$$\left.\frac{dy}{dx}\right|_{x=2} = 12(2)^3 - 24(2)^2 = 96 - 96 = 0$$

(c) Determine the nature of this stationary point, giving a reason for your answer. (2)

$$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 36(2)^2 - 48(2) = 144 - 96$$
$$= 48 > 0$$

Hence the stationary point is a minimum

6. Prove, from first principles, that the derivative of $3x^2$ is $6x$. (4)

$$f(x) = 3x^2$$
$$f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2$$

$$f'(x) = \lim_{h\to 0}\left[\frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{6xh + 3h^2}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{h(6x+3h)}{h}\right]$$

$$= \lim_{h\to 0}(6x+3h) = 6x$$

2. The curve C has equation

$$y = 2x^2 - 12x + 16$$

Find the gradient of the curve at the point P (5, 6). (4)

$$\frac{dy}{dx} = 4x - 12$$

$$\left.\frac{dy}{dx}\right|_{x=5} = 20 - 12 = 8$$

8 Prove that the function $f(x) = x^3 - 3x^2 + 15x - 1$ is an increasing function.

[6 marks]

Increasing when $f'(x) > 0$

$$f'(x) = 3x^2 - 6x + 15$$

$$= 3(x^2 - 2x) + 15$$

$$= 3[(x-1)^2 - 1] + 15$$

$$= 3(x-1)^2 + 12$$

$f'(x)$ has a minimum value of 12

$\therefore$ $f'(x) > 0$ for all values of x

Hence $f(x)$ is always increasing

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